Navier-Stokes 方程式(Navier-Stokes equations)

動量方程式如下表示時

$$\int_\Omega \frac{\partial \rho v_1}{\partial t} d\Omega = -\int \rho v \cdot v_1 d\Gamma + \int p_1 d\Gamma + \int b_1 d\Omega$$

$$\int_\Omega \frac{\partial \rho v_2}{\partial t} d\Omega = -\int \rho v \cdot v_2 d\Gamma + \int p_2 d\Gamma + \int b_2 d\Omega$$

$$\int_\Omega \frac{\partial \rho v_3}{\partial t} d\Omega = -\int \rho v \cdot v_3 d\Gamma + \int p_3 d\Gamma + \int b_3 d\Omega$$

當表面力與應力度間以下列關係式表示

$$p_1 = \sigma_{11} n_1 + \sigma_{12} n_2 + \sigma_{13} n_3$$
$$p_2 = \sigma_{21} n_1 + \sigma_{22} n_2 + \sigma_{23} n_3$$
$$p_3 = \sigma_{31} n_1 + \sigma_{32} n_2 + \sigma_{33} n_3$$

又因 $v = v_1 n_1 + v_2 n_2 + v_3 n_3$，利用發散定理將上 3 式右邊的邊界積分轉換成領域積分得

$$\int_\Omega \frac{\partial \rho v_1}{\partial t} d\Omega = -\int \left[\frac{\partial}{\partial x_1}\left(\rho v_1^{2}\right) + \frac{\partial}{\partial x_2}\left(\rho v_1 v_2\right) + \frac{\partial}{\partial x_3}\left(\rho v_1 v_3\right) \right] d\Omega + \int \left(\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} + \frac{\partial \sigma_{13}}{\partial x_3} \right) d\Omega + \int b_1 d\Omega$$

$$\int_\Omega \frac{\partial \rho v_2}{\partial t} d\Omega = -\int \left[\frac{\partial}{\partial x_1}\left(\rho v_1 v_2\right) + \frac{\partial}{\partial x_2}\left(\rho v_2^{2}\right) + \frac{\partial}{\partial x_3}\left(\rho v_2 v_3\right) \right] d\Omega + \int \left(\frac{\partial \sigma_{21}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{23}}{\partial x_3} \right) d\Omega + \int b_2 d\Omega$$

$$\int_\Omega \frac{\partial \rho v_3}{\partial t} d\Omega = -\int \left[\frac{\partial}{\partial x_1}\left(\rho v_1 v_3\right) + \frac{\partial}{\partial x_2}\left(\rho v_2 v_3\right) + \frac{\partial}{\partial x_3}\left(\rho v_3^{2}\right) \right] d\Omega + \int \left(\frac{\partial \sigma_{31}}{\partial x_1} + \frac{\partial \sigma_{32}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3} \right) d\Omega + \int b_3 d\Omega$$

考量連續方程式，得下 Navier-Stokes 方程式

$$\rho \left(\frac{\partial v_1}{\partial t} + v_1 \frac{\partial v_1}{\partial x_1} + v_2 \frac{\partial v_1}{\partial x_2} + v_3 \frac{\partial v_1}{\partial x_3} \right) = \frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} + \frac{\partial \sigma_{13}}{\partial x_3} + b_1$$

$$\rho \left(\frac{\partial v_2}{\partial t} + v_1 \frac{\partial v_2}{\partial x_1} + v_2 \frac{\partial v_2}{\partial x_2} + v_3 \frac{\partial v_2}{\partial x_3} \right) = \frac{\partial \sigma_{21}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{23}}{\partial x_3} + b_2$$

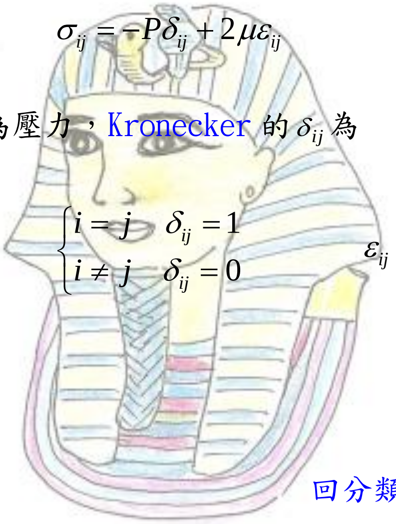
$$\rho\left(\frac{\partial v_3}{\partial t}+v_1\frac{\partial v_3}{\partial x_1}+v_2\frac{\partial v_3}{\partial x_2}+v_3\frac{\partial v_3}{\partial x_3}\right)=\frac{\partial \sigma_{31}}{\partial x_1}+\frac{\partial \sigma_{32}}{\partial x_2}+\frac{\partial \sigma_{33}}{\partial x_3}+b_3$$

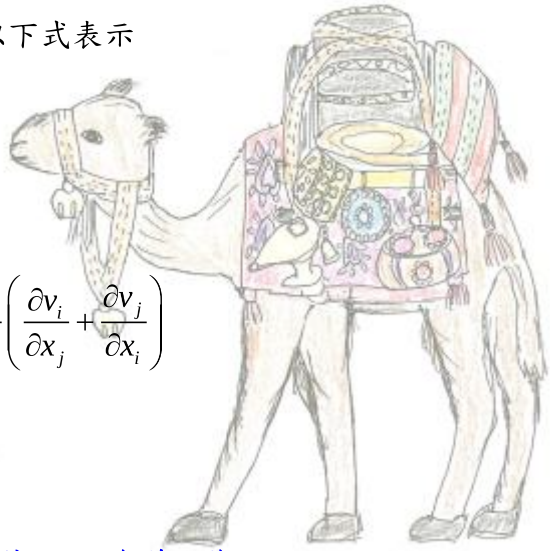
流體粘性係數為 μ 時，應力度可以下式表示

$$\sigma_{ij}=-P\delta_{ij}+2\mu\varepsilon_{ij}$$

P 為壓力，Kronecker 的 δ_{ij} 為

$$\begin{cases} i=j & \delta_{ij}=1 \\ i\neq j & \delta_{ij}=0 \end{cases} \qquad \varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)$$





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