Boundary Element Methods
M. Tanaka and Z. Yao (Editors)

Deformation of solitary wave in coastal zones

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In this study, generation,propagation and deformation of nonlinear waves were numerical simulated by means of boundary element method . The algorithm was based on the Largrange description and finite difference method to time to solve deformation problem of two dimensional wave. In our numerical model, wave-making scheme was used, therefore any type of desired wave could be generated, propagation of wave pass through underwater obstacle or wave run up in coastal zone was obtained. In this paper, first, the credibility of the numerical model was checked for the case that finite amplitude wave propagate and run up on a vertical wall, we found that the results were good. Secondly, soliton was simulated, time history of run up on a slope was shown, the case there has a submerge obstacle was also shown.

1.INTRODUCTION

Propagation and deformation of soliton in coastal zone was studied by previous scholars. Liu [1](1984) studied diffraction of solitary wave passing through semi-infinite thin barrier in both theory and experiment, Mei[2](1985) obtained coefficient of energy dissipation, reflection and transmission for soliton wave pass through a step by semi-experimental equation. Ouyama[3](1985) explored solitary wave set up on slope by boundary element method. Seabra-Santos , Renouard & Temperville[4](1987) introduced long-wave equations including curvature effects to describe the deformation and fission of a barotropic solitary wave passing over a shelf or isolated obstacle. Chang & Tang[5](1992) employed the finite element method with transient boundary fitted grid system to analyse solitary wave interacted by the submerged steplike shelf. In this study, generation and propagation of nonlinear waves were numerical simulated by means of boundary element method. The algorithm was based on Largrange description and finite difference method to time to solve deformation problem of two dimensional wave. In our numrical model, wave-making scheme was used, therefore any type of desired wave could be generated, propagation of wave psss through underwater obstacle or wave run up in coastal zone was obtained.

2. THEORETICAL ANALYSIS

As shown in Fig. 1, Cartesian coordinates are employed, the origin of which is located on water surface at rest with the z-axis vertically upwards. The fluid field is closed by a pseudo wave making boundary Γ_1, free water surface Γ_2 and impermeable sea bed Γ_3. We assumed that pseudo wave making boundary Γ_1 is sufficiently far away from coastal zone, wave scattering induced by undersea topography or obstacle can be neglected. The fluid is assumed to be inviscid, incompressible and flow is irrotational. Fluid motion has velocity potential $\Phi(x,z;t)$ which has to satisfy following Laplace equation

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial z^2} = 0 \quad (1)$$

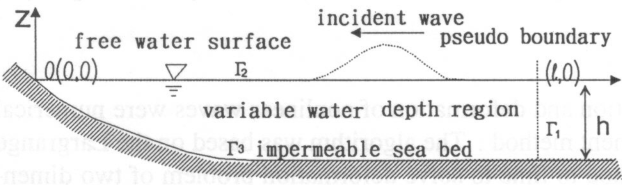


Fig. 1 Defintion sketch

2.1 Boundary conditions

(1) Boundary condition on pseudo wave making boundary Γ_1

Wave will be generated at pseudo-boundary Γ_1, it means that Γ_1 is a numerical wave-making paddle, any desired type of paddle can be simulated, but in this study piston type is used. Due to requirement of continuity between horizontal velocity of pseudo wave-making paddle U(t) and fluid flow, we obtain following relation.

$$\overline{\Phi} = \frac{\partial\Phi}{\partial\nu} = U(t) \quad (2)$$

Any desired wave can be simulated by input suitable U(t), finite amplitude wave and solitary wave are simulated in this paper. For finite amplitude wave, U(t) is expressed as

$$U(t) = -a\sigma\sin\sigma t \quad (3)$$

$$a = \zeta_0 \cdot \frac{sinhkh \cdot coshkh + kh}{2sinh^2kh} \quad (4)$$

Where $\sigma\,(=2\pi/T)$, ζ_0, k and T are angular wave frequency, wave amplitude, wave number and wave period to be generated, respectively.

To simulate the solitary wave , U(t) can be expressed by

$$U(t) = x_0 \cdot \omega \cdot sech^2[\omega(t-t_c)] \quad (5)$$

$$x_0 = h\sqrt{\frac{4\xi_0}{3(h+\xi_0)}} \quad (6)$$

$$\omega = \sqrt{gh}\sqrt{\frac{3}{4}\xi_0(1+\frac{\xi_0}{h})} \quad (7)$$

$$t_c = \pi/\omega \quad (8)$$

Where x_0 denotes the semistroke of wave making paddle and ξ_0 is the wave height of solitary wave to be generated.

(2) Boundary condition on free water surface

Assumed that air pressure on free water surface is constant, the boundary condition on the free water surface can be obtain from the kinematic and dynamic condition as

$$u = \frac{Dx}{Dt} = \frac{\partial \Phi}{\partial x} \tag{9}$$

$$w = \frac{Dz}{Dt} = \frac{\partial \Phi}{\partial z} \tag{10}$$

$$\frac{D\Phi}{Dt} + g\zeta - \frac{1}{2}[(\frac{\partial \Phi}{\partial x})^2 + (\frac{\partial \Phi}{\partial z})^2] = 0 \tag{11}$$

where D is the Lagrange differentiation, g is the acceleration of gravity and ζ is profile of free water surface.

(3) Boundary condition on the impermeable sea bed

Due to water particle velocity normal to impremeable seabed has to be null, we have

$$\frac{\partial \Phi}{\partial \nu} = 0 \tag{12}$$

where ν is outward normal on sea bed.

2.2 Integral equation

According to Green's second identity law, velocity potential $\Phi(x, z; t)$ at any point in the region can be expressed by velocity potential and its normal derivative on the boundary as

$$\Phi(x, z; t) = \frac{1}{2\pi} \int_\Gamma [\frac{\partial \Phi(\xi, \eta; t)}{\partial \nu} \ln \frac{1}{r} - \Phi(\xi, \eta; t) \frac{\partial}{\partial \nu} \ln \frac{1}{r}] ds \tag{13}$$

where $r = [(\xi - x)^2 + (\eta - z)^2]^{\frac{1}{2}}$.

When the inner point (x,z) approach to boundary point (ξ', η'), due to its singularity, velocity potential $\Phi(\xi, \eta; t)$ can be expressed as

$$\Phi(\xi', \eta', t) = \frac{1}{\pi} \int_\Gamma [\frac{\partial \Phi(\xi, \eta, t)}{\partial \nu} \ln \frac{1}{R} - \Phi(\xi, \eta, t) \frac{\partial}{\partial \nu} \ln \frac{1}{R}] ds \tag{14}$$

where $R = [(\xi - \xi')^2 + (\eta - \eta')^2]^{\frac{1}{2}}$

To proceed with numerical calculation, the boundaries Γ_1 through Γ_3 are divided into N_1 to N_3 discrete segments with linear elements, above equation can be written in a discretized form as

$$\begin{aligned} \Phi_i(\xi', \eta', t) + \frac{1}{\pi} \sum_{j=1}^{N} \int_{\Gamma_j} [\Phi_j(\xi, \eta, t) M_1 + \Phi_{j+1}(\xi, \eta, t) M_2] \frac{\partial}{\partial \nu} \ln \frac{1}{r} ds \\ = \frac{1}{\pi} \sum_{j=1}^{N} \int_{\Gamma_j} [\overline{\Phi}_j(\xi, \eta, t) M_1 + \overline{\Phi}_{j+1}(\xi, \eta, t) M_2] \ln \frac{1}{r} ds \end{aligned} \tag{15}$$

where $\overline{\Phi}_j = \partial\Phi_j/\partial\nu$; $\overline{\Phi}_{j+1} = \partial\Phi_{j+1}/\partial\nu$, and M_1, M_2 are the shape functions, $M_1 = (1-\chi)/2$, $M_2 = (1+\chi)/2$,χ is a local dimensionless coordinate.

Eq.(15)can be expressed in matrix form as

$$[\Phi] = [O]\,[\overline{\Phi}] \tag{16}$$

where $[\Phi]$ and $[\overline{\Phi}]$ are the potential function and its normal derivative on the boundaries, $[O]$ is a matrix relate to geometrical shape of boundary. The numerical scheme is discussed in detail by Chou(1983) [6]

To proceed above equations Gauss integration formula were used. For the propose of substituting boundary conditions into each boundaries, we rewrite Eq.(16) as follow

$$[\Phi_i] = [O_{ij}]\,[\overline{\Phi}_j] \quad , \quad i, j = 1 \sim 3 \tag{17}$$

2.3 Simultaneous equations

2.3.1 Initial conditions

The initial boundary conditions on each boundaries are summarized as follows

(1).Pseudo wave making boundary Γ_1

Requirement of continuity between horizontal velocity of pseudo wave-making paddle U(t) and fluid motion, we obtain

$$\overline{\Phi}_1^0 = \frac{\partial\Phi_1^0}{\partial\nu} = -U(0) \tag{18}$$

(2).Free water surface Γ_2

Assume the water surface is at rest at t=0, and the velocity potential is null ,i.e.

$$\Phi_2^0 = 0 \tag{19}$$

(3).Impermeable sea bed Γ_3

There is no flow exist on the direction normal to sea bed, give

$$\overline{\Phi}_3^0 = \frac{\partial\Phi_3^0}{\partial\nu} = 0 \tag{20}$$

where the superscript "0" indicates the time begin to simulation.

2.3.2 Finite Difference of related terms

The tangential derivative $(\partial\Phi_2/\partial s)_i$ on free water surface can be approximated as

$$
\left.\begin{aligned}
(\frac{\partial\Phi_2}{\partial s})_i &= (\frac{\Delta s_i}{\Delta s_{i+1}})\cdot\Phi_{2,i+1}/s' + (\Delta s_{i+1}-\Delta s_i)\cdot\Phi_{2,i}/s'' - (\frac{\Delta s_{i+1}}{\Delta s_i})\cdot\Phi_{2,i-1}/s' \\
s' &= \Delta s_{i+1}+\Delta s_i \\
s'' &= \Delta s_i\cdot\Delta s_{i+1}
\end{aligned}\right\} \quad (21)
$$

On the free surface, we have

$$
\left.\begin{aligned}
\frac{\partial\Phi_2}{\partial x} &= \frac{\partial\Phi_2}{\partial\nu}\sin\beta - \frac{\partial\Phi_2}{\partial s}\cos\beta \\
\frac{\partial\Phi_2}{\partial z} &= \frac{\partial\Phi_2}{\partial\nu}\cos\beta + \frac{\partial\Phi_2}{\partial s}\sin\beta
\end{aligned}\right\} \quad (22)
$$

where β denotes the angle which free water surface make with x-axis.

At k-th time step, the profile of free water surface is expressed by (x^k, z^k), from Eq.9 and Eq.10 we can evaluate that expressed by (x^{k+1}, z^{k+1}) at k+1-th time step as

$$x^{k+1} = x^k + (\frac{\partial\Phi_2^k}{\partial x})\Delta t \quad (23)$$

$$z^{k+1} = z^k + (\frac{\partial\Phi_2^k}{\partial z})\Delta t \quad (24)$$

where Δt denotes time difference interval.

From Eq.11 and Eq.22, velocity potential on free surface Φ^{k+1} at k+1-th time step can be approximately evaluted by

$$\Phi_2^{k+1} = \Phi_2^k + \frac{1}{2}[(\frac{\partial\Phi_2}{\partial s})^2 + (\frac{\partial\Phi_2}{\partial\nu})^2]^k\Delta t - gz^{k+1}\Delta t \quad (25)$$

Substituting Eq.2, Eq.12 and the above equation into Eq.17, we can obtain following simultaneous equations.

$$
\begin{bmatrix} \Phi_1 \\ \overline{\Phi}_2 \\ \Phi_3 \end{bmatrix}^{k+1} = \begin{bmatrix} I & -O_{12} & 0 \\ 0 & -O_{22} & 0 \\ 0 & -O_{32} & I \end{bmatrix}^{-1} \begin{bmatrix} O_{11} & 0 & O_{13} \\ O_{21} & -I & O_{23} \\ O_{31} & 0 & O_{33} \end{bmatrix} \begin{bmatrix} \overline{\Phi}_1 \\ \Phi_2 \\ \overline{\Phi}_3 \end{bmatrix}^{k+1} \quad (26)
$$

2.3.3 Time iteration process

1. At the beginning time of simulation, substituting Eq.18 to Eq.20 into Eq.17, normal derivative of velocity potentials on water surface $\partial\Phi_2^0/\partial\nu$, velocity potentials on pseudo wave making paddle Φ_1^0 and velocity potentials on sea bed Φ_3^0 can be obtained.

2. From Eq.21 tangential derivative of velocity potentials on water surface $\partial\Phi_2^0/\partial\nu$ are given.

3. Profile of water surface (x^1, z^1) for next time step can be obtained from Eq.23 and Eq.24.

4. The velocity potentials on free water surface for next time step will be given by Eq.25.

5. At t=Δt time step, matrix $[O]$ in Eq.17 will be carried out again under new profile of water surface obtained by procedure 3 and new position of pseudo wave-making paddle.

6. Substituting velocity potentials on new water surface given by procedure 4, horzintal velocity U(t) of pseudo wave-making paddle given by Eq.2 and boundary condition on sea bed given by Eq.12 into Eq.26, the normal derivative of velocity potentials $\partial\Phi_2^1/\partial\nu$ on water surface, velocity potentials Φ_1^1 on pseudo wave-making paddle and velocity potentials Φ_3^1 on sea bed for t=Δt time step can be obtained.

7. Repeating above procedure 2 to 6,the time history of generation,propagation and deformation of wave can be simulated.

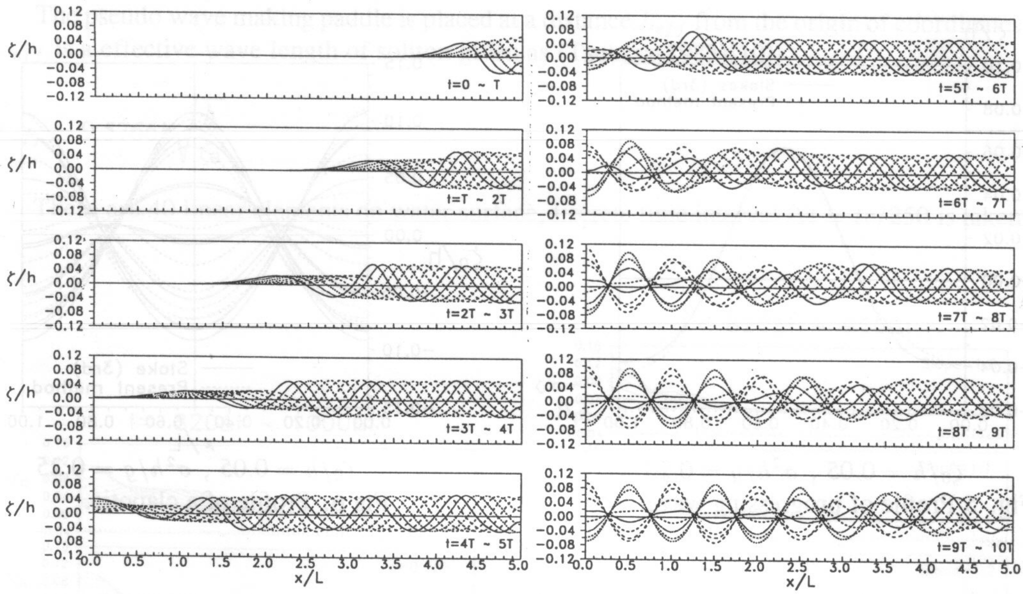
3. NUMERICAL COMPUTATIONS AND DISCUSSIONS

3.1 Finite amplitude wave

For the propose to confirm the credibility of this numerical model, finite amplitude wave is simulated with incident wave amplitude $\zeta_0 = 0.05h$ and dimensionless angular frequencies $\sigma^2 h/g$=0.25, 0.5, 1.0, 1.25. The pseudo wave making paddle is placed at a distance of 5 times wave length from the origin of coordinates. For the case of vertical wall there are 32 linear elements for each wave length, i.e. $\Delta s = L/32$ and time discrete interval $\Delta t = T/160$ are taken, but for cases of inclined slope $\Delta s = L/40$ and $\Delta t = T/400$ are used. Fig.2, Fig.3 and Fig.4 show the time history of sinuous wave generated, propaged and run up a vertical wall and a inclined slope with slopes 1:0 to 1:1. Fig.5 shows the profiles of a progressing wave without the effects reflection induced by the vertical wall, it is compared to the 3rd order Stokes wave derived by Skjelbreia(1959)[7], good agreement is found. Fig.6 shows the time history of wave profile at the time incident wave is reflected by the wall, i.e. clapotis is formed, it is compared to the theoretical results given by Tadjbaksh(1960)[8], good agreement is found also. From Fig.4 we can find the characteristic of finite amplitude wave become stronger when steepness of profile of incident wave become larger.

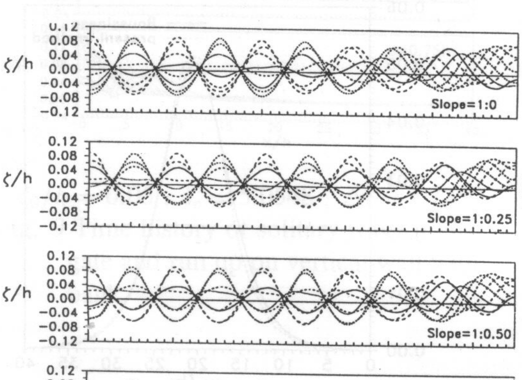
3.2 Solitary wave

Fig.7 shows soliton run up-down a vertical wall with incident wave height $\zeta_0/h = 0.05$. Fig.8 shows the profile of soliton at the time that the reflecting effect induced by the wall is not appeared yet, compares it to theoritical results given by Boussinesq(1872)[9], good agreement is also found. Fig.9 shows the time history of water surface profile which a submerge bank is fixed in front of the vertical wall, the layout is shown in figure. Fig.10 show that of a incline wall.



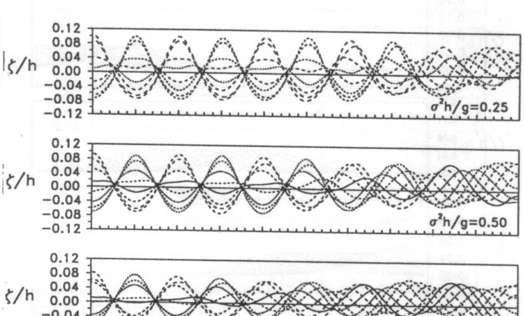
$\zeta_0/h = 0.05$, $\sigma^2 h/g = 0.25$, $\Delta t = T/160$, $\Delta s = L/32$

Fig. 2 Time history of wave propagate and run up on a vertical wall.



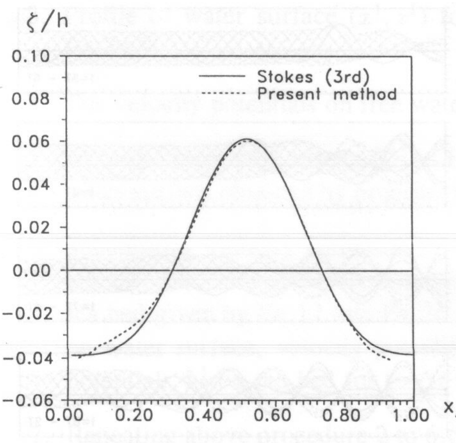
$\zeta_0/h = 0.05, \sigma^2 h/g = 0.25, \Delta t = T/400, \Delta s = L/40$

Fig. 3 Profiles of standing wave propagate on slopes.



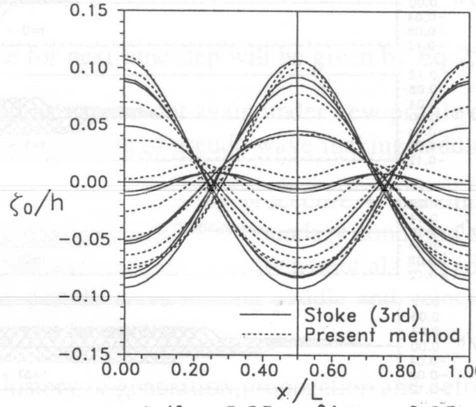
$\zeta_0/h = 0.05, \Delta t = T/160, \Delta s = L/32$

Fig 4. Comparison of standing wave between different cases of difference dimensionless frequencies.



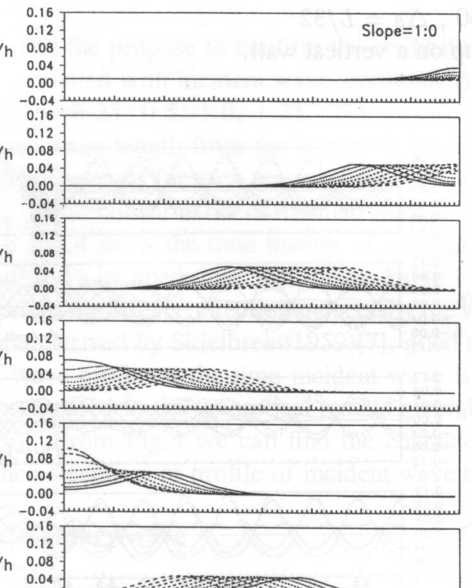
$\zeta_0/h = 0.05$, $\sigma^2 h/g = 0.25$

Fig. 5 Profile of progressing wave.



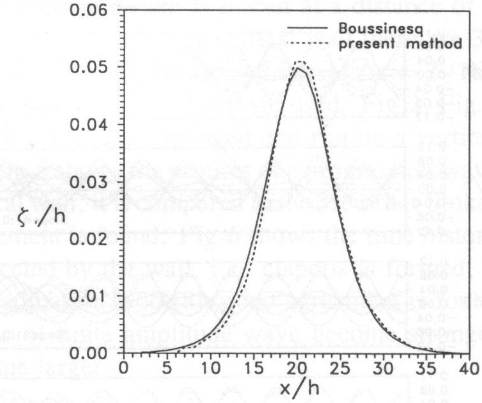
$\zeta_0/h = 0.05$, $\sigma^2 h/g = 0.25$

Fig 6. Profile of a clapotis.



$\zeta_0/h = 0.05$, $t_c = 5.0548$, $\Delta t = t_c/250$

Fig. 7 Time history of solitary wave propagate and reflect on a vertical wall.



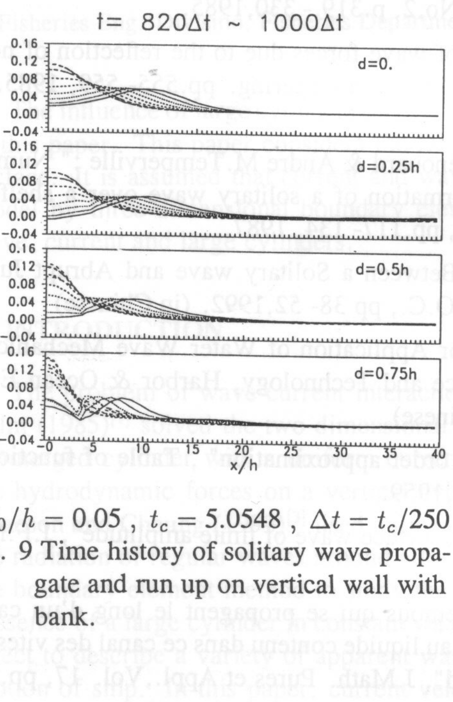
$\zeta_0/h = 0.05$, $t_c = 5.0548$

Fig 8. Profile of a solitary wave.

The pseudo wave making paddle is placed at a distance L_{eff} from the origin of coordinates. L_{eff} is a effective wave length of soliton given as (Tsukasa(1983)[10])

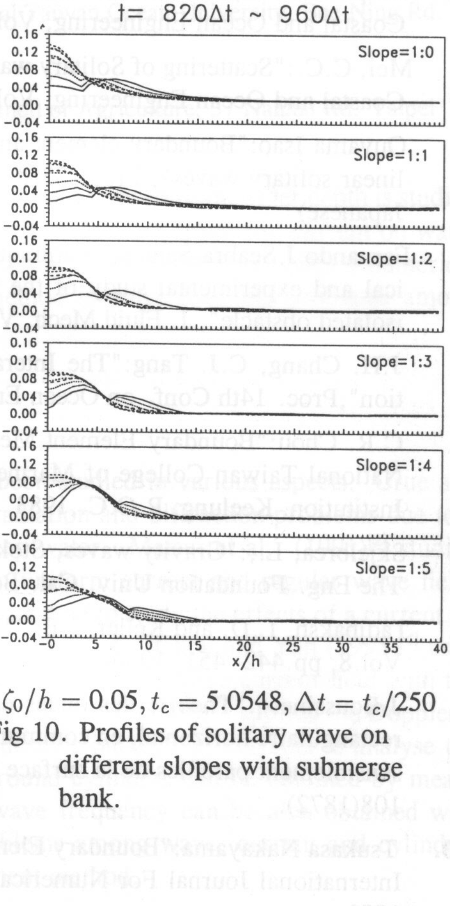
$$L_{eff} = 9.5766h\sqrt{\frac{h}{\zeta_0}} \tag{27}$$

There are 40 linear elements on water surface,discrete time interval $\Delta t = tc/250$ is taken.



$\zeta_0/h = 0.05$, $t_c = 5.0548$, $\Delta t = t_c/250$

Fig. 9 Time history of solitary wave propagate and run up on vertical wall with bank.



$\zeta_0/h = 0.05, t_c = 5.0548, \Delta t = t_c/250$

Fig 10. Profiles of solitary wave on different slopes with submerge bank.

4.CONCLUSIONS

Algorithm in this article using boundary element technique based on Largrange description and finite difference method to time is applied to analyse two dimensional nonlinear sinuous wave and solitary wave problem as a wave making problem. When soliton is simulated, we found that the stability of this scheme is good for the slope of wall less then 1:5. On the other hand, for sinuous wave , the slope of wall greater than 1:1 numerical unstability will be occured.

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