Boundary Element Methods
M. Tanaka and Z. Yao (Editors)

Wave-current interaction with large cylinders in constant water depth

C. R. Chou* S. S. Yan* and W.Y. Han**

*Department of Harbor and River Engineering, National Taiwan Ocean University, Bee-Ning Rd. 2, Keelung, China

** Fisheries Eng. Divisino, Fisheries Department, Council of Agriculture, 37 Nanhai Rd. Taipei

The influence of large cylinders on a wave-current field with constant water depth is studied in this paper. This paper considers the Doppler's effect between wave and current on water surface. It is assumed that current and wave amplitude are small, and provides a numerical model by three-dimensional boundary element method to solve coupling problems among wave, current and large cylinders.

1. INTRODUCTION

The problem of wave-current interaction has been studied in various aspects. Grue and Palm (1985)[1] solved the two-dimensional wave radiation and diffraction problems due to a submerged cylinder with a uniform current in deep water. Matsui et al. (1991)[2] studied the hydrodynamic forces on a vertical cylinder in uniform current and regular wave field. Isaacson and Cheung (1993)[3] used a time-domain method to study the effects of a current on the radiation of regular waves around a two-dimensional body. Chou and Yan (1995)[4] used the boundary element method to study the coupling problems in wave-current field with the presence of a large cylinder in constant water depth. Newman (1978)[5] applied the Doppler's effect to describe a variety of apparent wave frequencies induced by currents to analyse the motion of ship. In this paper, current velocity around cylinders can be obtained by means of the boundary element method, the apparent wave frequency can be also obtained with Doppler's effect. Successively, the coupling problems among wave, current and cylinders can be solved by three-dimensional boundary element method.

2. THEORETICAL ANALYSIS

Figure 1 schematically shows wave-current field in the presence of large cylinders. The radius of cylinder is a, and water depth is h. A Cartesian coordinate system is employed, the water surface at rest is located at o-xy with z-axis vertically upwards. As shown in figure 1, the flow field is divided into two regions by a pseudo-boundary Γ_1. Region I is an outer region bounded by Γ_1 and infinite-field boundary Γ_∞, Region II is a region bounded by Γ_1, surfaces of cylinders, free water surface and impermeable seabed. The presence of cylinders affects waves in both regions. However, if the pseudo-boundary Γ_1 is sufficiently far from cylinders ($\geq L/2$), the wave scattering and current disturbance induced by them can be neglected in Region I.

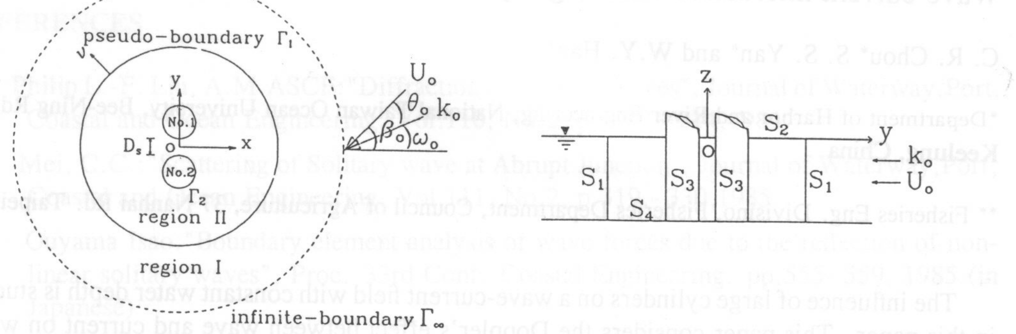


Fig. 1 Definition sketch of cylinders in wave-current field

It is assumed that the fluid inviscid, incompressible, and flow is irrotational. Therefore, there exists a velocity potential that satisfies the Laplace equation in both regions. We consider a steady current U_o and small amplitude waves having angular frequency σ_o ($= 2\pi/T$, T is wave period) and amplitude ζ_o, incident from infinite-field at angles of β_o and ω_o, respectively, against x-axis. If current velocity U_o and wave amplitude ζ_o are assumed to be small, the potential Φ of wave-current field can be expressed as a couple of current potential Φ^C and wave potential Φ^W which including the effects of current[3][5] as

$$\Phi(x, y, z; t) = \Phi^C(x, y) + \Phi^W(x, y, z; t) \tag{1}$$

where Φ, Φ^C, Φ^W have to satisfy Laplace equation.

2.1. Velocity potential of current in Region I

The velocity potential of current $\varphi^{(1)}$ in Region I can be expressed as a sum of a steady velocity potential φ^o and a disturbance velocity potential φ^* induced by the presence of cylinders as

$$\varphi^{(1)} = \varphi^o + \varphi^* \tag{2}$$

where $\varphi^o = -U_o(x\cos\beta_o + y\sin\beta_o)$, and φ^* should be zero on far field which includes the effects of cylinders and satisfy the following Laplace equation.

$$\frac{\partial^2\varphi^*}{\partial x^2} + \frac{\partial^2\varphi^*}{\partial y^2} = 0 \tag{3}$$

Applying the Green's theorem, potential $\varphi^*(x, y)$ for any point in Region I can be calculated by following integral equation,

$$c\,\varphi^*(x, y) = \frac{1}{2\pi}\int_{\Gamma_1}\left[\frac{\partial\varphi^*(\xi,\eta)}{\partial\nu}\ln\frac{1}{r} - \varphi^*(\xi,\eta)\frac{\partial}{\partial\nu}(\ln\frac{1}{r})\right]ds \tag{4}$$

where $\varphi^*(\xi,\eta)/$ and $\partial\varphi^*(\xi,\eta)/\partial\nu(=\overline{\varphi}^*)$ are velocity potential and its normal derivative with ν the local normal coordinate to boundary taken outwards, and $r=\sqrt{(x-\xi)^2+(y-\eta)^2}$ is distance between a point under consideration and boundary. The factor c is equal to unity within boundary, but will be 1/2 on smooth boundaries.

In the numerical analysis, the boundary Γ_1, where $c=1/2$, is discretized into N_1 segments with constant line element. Eq. (4) is rewritten in a matrix form as

$$\{\varphi^*\}=\{k^*\}\{\overline{\varphi}^*\} \tag{5}$$

where $\{k^*\}$ is a coefficient matrix related to the geometric shape of Γ_1. The numerical scheme is discussed in detail by Chou (1983)[6].

2.2. Velocity potential of current in Region II

The velocity potential $\varphi^{(2)}$ is affected by cylinders in this region, and satisfies Laplace equation. Applying the Green's theorem, velocity potential $\varphi^{(2)}$ can be calculated from the following integral equation.

$$c\,\varphi^{(2)}(x,y)=\frac{1}{2\pi}\int_{\Gamma_1+\Gamma_2}\left[\frac{\partial\varphi^{(2)}(\xi,\eta)}{\partial\nu}\ln\frac{1}{r}-\varphi^{(2)}(\xi,\eta)\frac{\partial}{\partial\nu}(\ln\frac{1}{r})\right]ds \tag{6}$$

As before, Eq. (6) can be expressed as form of Eq. (5)

$$\{\varphi^{(2)}\}=\{k\}\{\overline{\varphi}^{(2)}\} \tag{7}$$

The numerical scheme is discussed in detail by Chou (1983)[6].

The flow is null in normal direction to the surfaces of cylinders, free water surface and impermeable seabed. Requirement of mass and energy flux continuity between Region I and Region II at the pseudo-boundary Γ_1 leads to the following relation.

$$\overline{\varphi}^{(1)}=\overline{\varphi}^{(2)} \tag{8}$$

$$\varphi^{(1)}=\varphi^{(2)} \tag{9}$$

To facilitate substitution of the boundary conditions, we rewrite the governing equation, Eq. (7) into the following form.

$$\left\{\varphi_i^{(2)}\right\}=\{k_{ij}\}\left\{\overline{\varphi}_j^{(2)}\right\}\quad(i,j=1,2) \tag{10}$$

Substituting the boundary conditions into Eq.(10) and a little algebra leads to

$$\left\{\overline{\varphi}_1^{(2)}\right\}=\{k_{11}-k^*\}^{-1}\left\{\varphi^0-k^*\overline{\varphi}^0\right\} \tag{11}$$

Substituting Eq. (11) into Eq. (10), velocity potential $\varphi_1^{(2)}$ and $\varphi_2^{(2)}$ on boundaries Γ_1 and Γ_2 can be obtained.

2.3. Distributions of current velocity within Region II

Once the velocity potential $\varphi^{(2)}$ and its normal derivative $\overline{\varphi}^{(2)}$ of current are obtained, the current velocity at any inner point in region II can be derived as

$$u = \frac{1}{2\pi}\sum_{j=1}^{N_1+N_2}\left\{\overline{\varphi}_j^{(2)}\left(\frac{x-x_j}{r^2}\right) - \varphi_j^{(2)}\left[\frac{-\nu_x}{r^2} + \frac{2(x-x_j)[(x-x_j)\nu_x + (y-y_j)\nu_y]}{r^4}\right]\right\} ds_j \quad (12)$$

$$v = \frac{1}{2\pi}\sum_{j=1}^{N_1+N_2}\left\{\overline{\varphi}_j^{(2)}\left(\frac{y-y_j}{r^2}\right) - \varphi_j^{(2)}\left[\frac{-\nu_y}{r^2} + \frac{2(y-y_j)[(x-x_j)\nu_x + (y-y_j)\nu_y]}{r^4}\right]\right\} ds_j \quad (13)$$

where ν_x and ν_y are the components of normal vector ν in directions of x and y on boundary.

2.4. Wave velocity potential

It is assumed that current velocity and wave amplitude are small. Therefore, wave velocity potential $\Phi^W(x,y,z;t)$ which including the current effects can be expressed as follows[1]

$$\Phi^W(x,y,z;t) = \frac{g\zeta_o}{\sigma_o}\,\phi(x,y,z)\,e^{-i\sigma t} \quad (14)$$

where g is acceleration of gravity, ζ_o is incident wave height, i is imaginary unit, t is time, and wave number k_o satisfies the following dispersion relation.

$$\frac{\sigma_o^2 h}{g} = k_o h \tanh k_o h \quad (15)$$

The apparent frequency σ is angular frequency of wave affected by current, its means that Doppler's effect is considered by following relation

$$\sigma = \sigma_o + \vec{k}\cdot\vec{U} \quad (16)$$

where $|\vec{k}| = k_o$, $|\vec{U}|(=\sqrt{u^2+v^2})$ is the current velocity around cylinders. The potential function ϕ satisfies following Laplace equation.

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \quad (17)$$

2.4.1. Wave potential function in Region I

If the pseudo-boundary Γ_1 is sufficiently far away from cylinders, wave scattering induced by cylinders can be neglected in this region. The potential function $\phi^{(1)}(x,y,z)$ for Region I can be expressed as

$$\phi^{(1)}(x,y,z) = [f^o(x,y) + f^*(x,y)]\frac{\cosh k_o(z+h)}{\cosh k_o h} \quad (18)$$

where $f^o(x,y)$ is incident wave potential function, and $f^*(x,y)$ is diffracted wave potential function induced by the presence of cylinders.

Substituting Eq. (18) into Eq. (17), we obtain potential function f^*, which satisfies following Helmholtz equation.

$$\frac{\partial^2 f^*}{\partial x^2} + \frac{\partial^2 f^*}{\partial y^2} + k_o^2 f^* = 0 \quad (19)$$

On the far-field boundary, Sommerfeld radiation condition must be satisfied, and $f^*(x,y)$ should be zero. Applying the Green's theorem, potential function f^* for any point in Region I can be calculated from the following integral equation

$$c\, f^*(x,y) = \frac{i}{4}\int_{\Gamma_1}\left\{\overline{f}^*(\xi,\eta)H_o^{(1)}(k_o r) - f^*(\xi,\eta)\frac{\partial}{\partial\nu}[H_o^{(1)}(k_o r)]\right\} ds \tag{20}$$

where $f^*(\xi,\eta)$ is potential function specified by geometric shape of boundaries, $\overline{f}^*(\xi,\eta)(=\partial f^*(\xi,\eta)/\partial\nu)$ is its normal derivative with ν the local normal coordinate to boundary taken outwards, $H_o^{(1)}$ is zeroth order Hankel function of first kind. Accordingly stated in Section 2.1, Eq. (20) can be rewritten in a matrix form as

$$\{F^*\} = \{K^*\}\left\{\overline{F}^*\right\} \tag{21}$$

where $\{K^*\}$ is a coefficient matrix related to the geometric shape of Γ_1. The numerical scheme is discussed in detail by Chou (1983, 1993)[6][7].

2.4.2. Wave potential function in Region II

Region II is bounded by pseudo-boundary surface S_1, free water surface S_2, surfaces of cylinders S_3 and seabed S_4 (let $S = S_1 + S_2 + S_3 + S_4$). According to Green's second identity law, potential function $\phi^{(2)}(x,y,z)$ at any point within Region II can be determined by following integral equation

$$c\,\phi^{(2)}(x,y,z) = \frac{1}{4\pi}\int_S\left[\overline{\phi}^{(2)}(\xi,\eta,\zeta)\frac{1}{R} - \phi^{(2)}(\xi,\eta,\zeta)\frac{\partial}{\partial\nu}\frac{1}{R}\right] dA \tag{22}$$

where $R = \sqrt{(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2}$. As before, c is unity for point inside the region and is equal to 1/2 on boundaries.

To proceed with numerical calculation, surfaces of boundaries S_1 through S_4 are divided into N_1 through N_4 discrete segments with constant plane elements. For the case that $c = 1/2$, Eq. (22) is readily expressed as

$$\{\phi^{(2)}\} = \{K\}\left\{\overline{\phi}^{(2)}\right\} \tag{23}$$

As before, the numerical scheme is discussed in detail by Chou (1983, 1993)[6][7].

2.4.3. Boundary conditions

(1) Boundary condition on free water surface

The free surface condition of coupling wave-current field is given by linear wave theory[5] as follows,

$$\frac{\partial^2\Phi}{\partial t^2} + g\frac{\partial\Phi}{\partial z} = 0\ ,\ z = 0 \tag{24}$$

substituting Eq. (1) and Eq. (14) into Eq. (24), which can be rewritten in the following form as

$$\frac{\partial\phi^{(2)}}{\partial z} = \frac{\sigma^2}{g}\phi^{(2)}\ ,\ z = 0 \tag{25}$$

where $\sigma = \sigma_o + \vec{k}\cdot\vec{U}$.

(2) Boundary conditions on surfaces of cylinders and impermeable seabed

The flow is null in normal direction to surfaces of cylinders impermeable seabed, i.e.,

$$\overline{\phi}^{(2)} = 0 \tag{26}$$

(3) Boundary condition on pseudo-boundary

Requirement of mass and energy flux continuity between region I and region II at pseudo-boundary surface S_1 leads to the following relation.

$$\overline{\phi}^{(1)} = \overline{\phi}^{(2)} \tag{27}$$

$$\phi^{(1)} = \phi^{(2)} \tag{28}$$

2.4.4. Equation system

To facilitate substitution of the boundary conditions into Eq. (23), we rewrite it into the following form,

$$\left\{\phi_j^{(2)}\right\} = \{K_{ij}\}\left\{\overline{\phi}_j^{(2)}\right\} \qquad (i, j = 1 \sim 4) \tag{29}$$

By dividing the surface of pseudo-boundary S_1 into M segments vertically and N segments horizontally, applies boundary conditions of Eqs. (27) and (28) to obtain the relation between potential function and its normal derivative on pseudo-boundary S_1 as follow.

$$\{\phi_1\} = \{R\}\left\{F^o - K^*\overline{F}^o\right\} + C\{R\}\{K^*\}\{Q\}\left\{\overline{\phi}_1\right\} \tag{30}$$

where $C = k_o/N_o \sinh k_o h, (N_o = 1/2 + k_o h/\sinh 2k_o h)$, and the coefficient matrixes $\{R\}$ and $\{Q\}$ are diccussed in detail by Chou[6][7].

Substitution of Eqs. (25), (26) and (30) into Eq. (29) and a little algebra lead to

$$\begin{Bmatrix} K_{11} - CRK^*Q & \frac{\sigma^2}{g}K_{12} & 0 \\ K_{21} & \frac{\sigma^2}{g}K_{22} - I & 0 \\ K_{31} & \frac{\sigma^2}{g}K_{32} & -I \end{Bmatrix} \begin{Bmatrix} \overline{\phi}_1^{(2)} \\ \phi_2^{(2)} \\ \phi_3^{(2)} \end{Bmatrix} = \begin{Bmatrix} R[F^o - K^*\overline{F}^o] \\ 0 \\ 0 \end{Bmatrix} \tag{31}$$

By solving above equation, the derivative of potential functions on boundary S_1 and potential functions on boundaries S_2 and S_3 are obtained.

2.5. Wave forces and wave height distributions

The pressure throughout fluid can be evaluated by Bernoulli's equation, current velocity is assumed to be small and its squared term $|\nabla\Phi^c|^2$ can be neglected. We obtain the dynamic pressure (without static pressure) of first order as

$$p = -\rho\left[\frac{\partial\Phi^w}{\partial t} + \nabla\Phi^c \cdot \nabla\Phi^w\right] \tag{32}$$

where ρ is fluid density.

Substituting Eq. (14) into Eq. (32), taking integration around the surface of each cylinder yields total forces as

$$F = \frac{\rho g \zeta_o}{\sigma_o} e^{-i\sigma t} \int_{S_3} [-i\sigma\phi^{(2)} + \nabla\varphi^{(2)} \cdot \nabla\phi^{(2)}]\ \nu\ dA \tag{33}$$

From Eq. (33), the profile of water surface can be expressed as

$$\zeta = -\frac{1}{g}\left[\frac{\partial \Phi^{w}}{\partial t} + \nabla\Phi^{c}\cdot\nabla\Phi^{w}\right] \tag{34}$$

Substituting Eq. (14) into equation (34), the wave height ratio, K_d, in Region II, can be calculated by

$$K_d = \frac{1}{\sigma_o}\left|i\sigma\phi - \nabla\varphi^{(2)}\cdot\nabla\phi^{(2)}\right| \tag{35}$$

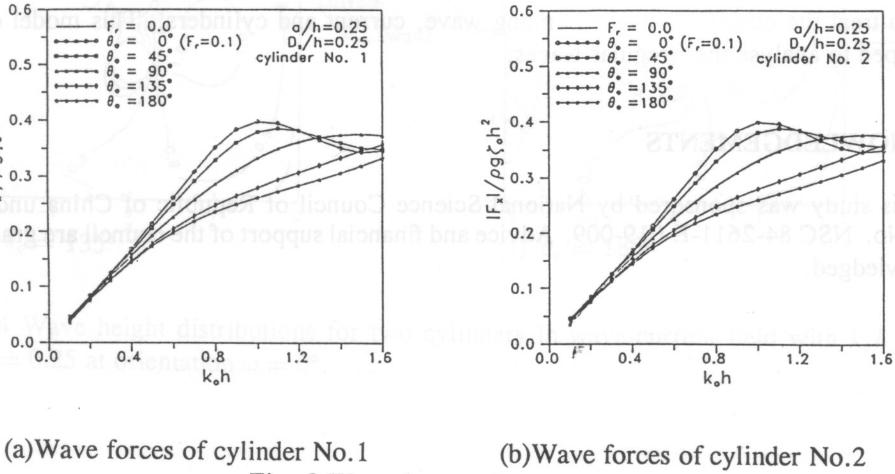
3. NUMERICAL RESULTS

The case of two cylinders within wave-current field is studied in this paper. The radius of cylinder a is $0.25h$, and distance D_s between two cylinders is $0.25h$. The current velocity is represented by a Froude number F_r $(= U_o/\sqrt{gh})$ which is 0.1. To facilitate the computations, the wave is propagating on opposite direction of x-axis, and wavenumber k_oh is varied from 0.1 to 1.6, and θ_o $(= \beta_o - \omega_o)$ 0°, 45°, 90°, 135° and 180° are taken.

In numerical analysis, the boundary surfaces were divided into 1412 discrete areas with constant element ($N_1 = 320$, $N_2 = 540$, $N_3 = 240$, $N_4 = 312$, and $M = 4$). The wave forces on those cylinders and distribution of wave heights for $k_oh = 1.0$ are presented.

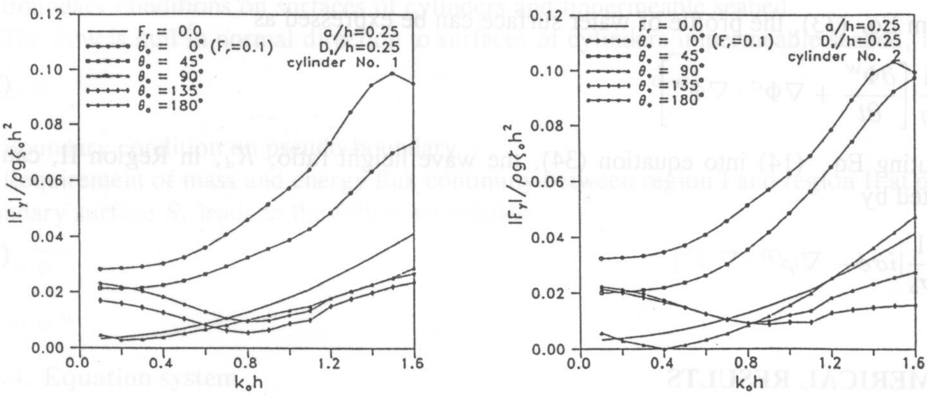
3.1. Wave forces

As shown in Figure 2, the x-component wave forces is reduced with θ_o is increased in the lower wave numbers ($k_oh < 1.2$). For the cases of $\theta = 0°$ and 45°, wave forces on each cylinder is greater than that in the absence of current, but for the cases of $\theta = 135°$ and 180°, it is less than that in the absence of current. Figure 3 shows the wave forces of y-component on each cylinder which is less than that of x-cmoponent.



(a)Wave forces of cylinder No.1 (b)Wave forces of cylinder No.2

Fig. 2 Wave forces of x-component



(a)Wave forces of cylinder No.1 (b)Wave forces of cylinder No.2

Fig. 3 Wave forces of y-component

3.2. Wave height distributions

Figures 4 is the distribution of wave heights in Region II for incident waves with $k_o h = 1.0$, and $F_r = 0.1$. As shown in figure, wave heights in the front of cylinders is greater than that behind cylinders induced by the sheltering effect. When current and wave are propagating in the same or opposite direction (i.e., $\theta_o = 0°$ or $180°$), the distribution of wave heights are symmetrical to x-direction. For the cases of $\theta_o = 45°$, $90°$ and $135°$, the distribution of wave heights become unsymmetrical is due to the influce of Doppler's effect which we have considered in our numerical model.

4. CONCLUSIONS

This paper considers the variety of wave frequency at any point on free water surface. In other words, the Doppler's effect is applied to this paper, and boundary element method is used to treat the coupling problem among wave, current and cylinders. This model can be developed to analyse the irregular waves.

ACKNOWLEDGEMENTS

This study was sponsored by National Science Council of Republic of China under the grant No. NSC 84-2611-E-019-009. Advice and financial support of the council are gratefully acknowledged.

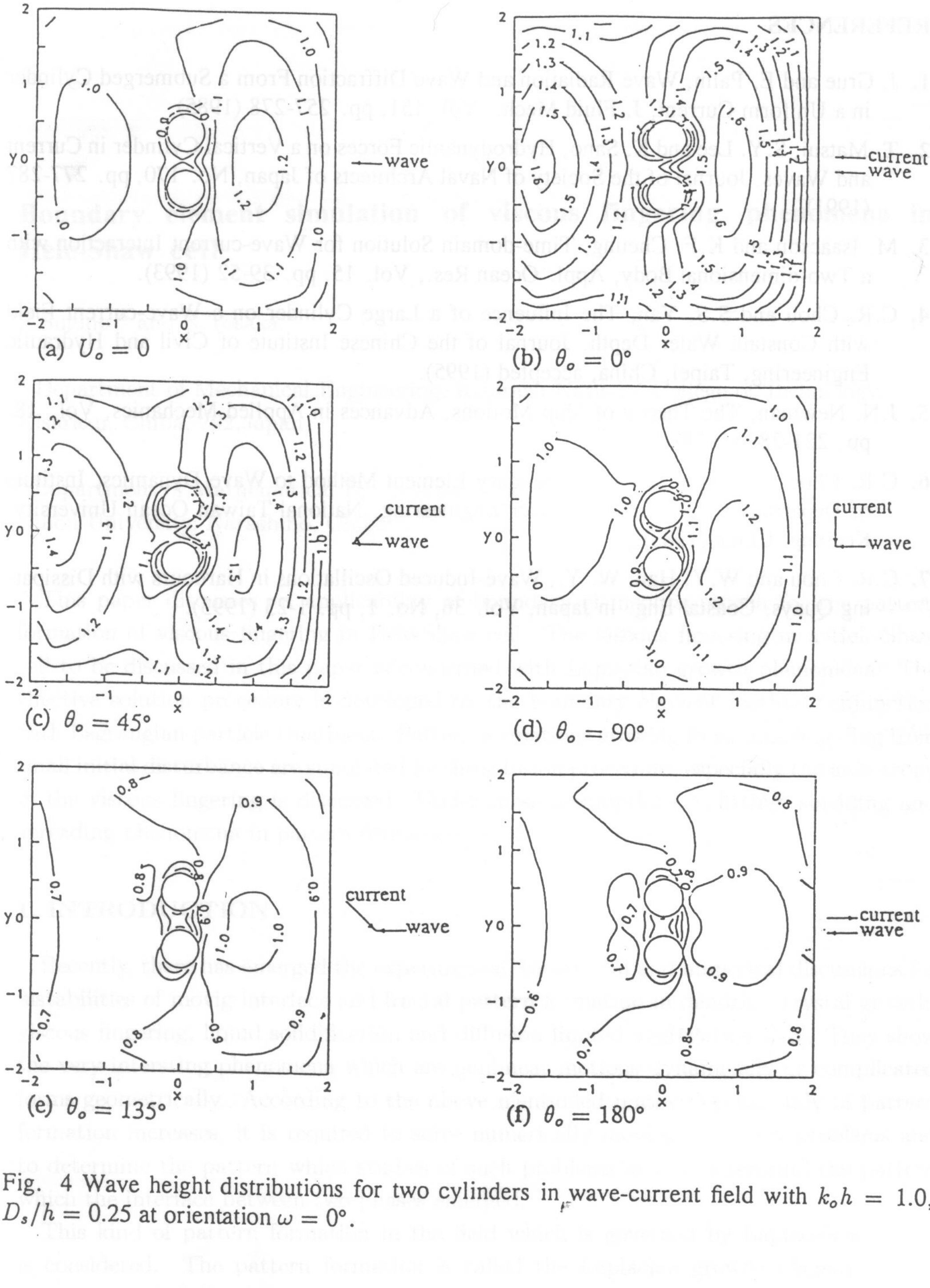


Fig. 4 Wave height distributions for two cylinders in wave-current field with $k_o h = 1.0$, $D_s/h = 0.25$ at orientation $\omega = 0°$.

REFERENCES

1. J. Grue and E. Palm, Wave Radiation and Wave Diffraction From a Submerged Cylinder in a Uniform Current, J. Fluid Mech., Vol. 151, pp. 257-278 (1985).

2. T. Matsui, S.Y. Lee and K. Sano, Hydrodynamic Forces on a Vertical Cylinder in Current and Waves, Journal of the Society of Naval Architects of Japan, No. 170, pp. 277-287 (1991).

3. M. Isaacson and K.F. Cheung, Time-domain Solution for Wave-current Interaction with a Two-dimensional Body, Appl. Ocean Res., Vol. 15, pp. 39-52 (1993).

4. C.R. Chou and S.S. Yan, The Influence of a Large Cylinder on a Wave-current Field with Constant Water Depth, Journal of the Chinese Institute of Civil and Hydraulic Engineering, Taipei, China, accepted (1995).

5. J.N. Newman, The Theory of Ship Motions, Advances in Applied Mechanics, Vol. 18, pp. 221-282 (1978).

6. C.R. Chou, The Application of Boundary Element Method to Wave Dynamics, Institute of Department of Harbor and River Engineering, National Taiwan Ocean University, Keelung, China (1983).

7. C.R. Chou and W.Y. Han, W. Y., Wave-Induced Oscillations in Harbours with Dissipating Quays, Coastal Eng. in Japan, Vol. 36, No. 1, pp. 1-23 (1993).